Discussion regarding Lauren William’s Amplituhedron day talk

Discussant: Karen Yeats
Combinatorial forms of positroids.

Decorated permutoheds

\[ \text{J diagram} \]

Grassmann necklace

\[ I_{i+1} = \left\{ (I_i \setminus \{i + 1\}) \cup \{j\} : i \in I_i \right\} \]

\[ I_i \quad \text{and} \quad I_{i+1} \]

\[ \begin{align*}
I_1 &= 146 \\
I_2 &= 246 \\
I_3 &= 456 \\
I_4 &= 456 \\
I_5 &= 146 \\
I_6 &= 145
\end{align*} \]
More combinatorial forms of positroids.

\( I_i \in \mathbb{R}^{k \times k} \) for some \( k \geq 0 \) and \( \lambda \in \mathbb{N}^{k+1} \).
Where does T-duality come from?

- Duality is in the sense of an equivalence of theories (string theories or QFTs esp. conformal field theories).
- Prototypically, it inverts the radius (of space-time, or of the torus fibres of a bundle whose total space is space-time, ...).
- In any case it is a transformation of the underlying space (and associated data) that give the same physics.
- People most often say T stands for target space (see point 3), alternately torus (see point 2) and others.
- It is important in string theory because in the 90s people realized certain string theories were related by T-duality.
- Precise versions are studied as pure differential geometry, etc.
- From the path integral, rewrite to integrate over auxiliary fields, then integrate the other way.
T-duality and the amplituhedron.

Williams and collaborators defined a combinatorial T-duality and proved many interesting properties of it, as we’ve just heard about. Why is this T-duality?

- In the Grassmannian context, T-duality maps between twistors and momentum twistors.
- One manifestation is the amplitude/Wilson loop duality.
- It maps between BCFW cells, $4k$-dimensional cells of $\text{Gr}_{k,n}^+$ conjecturally triangulating the amplituhedron $A_{n,k,4}$ and $2n - 4$-dimensional cells of $\text{Gr}_{k+2,n}^+$ conjecturally triangulating the momentum amplituhedron $\mathcal{M}_{n,k,4}$.
- Moving from 4 to 2, $\mathcal{M}_{n,k,2}$ is a kind of dual of $\Delta_{k+1,n}$.
- The combinatorial T duality is the corresponding map of cells. (Need an extra shift $\sigma(\hat{\pi}(i)) = \pi(i - 1) - 1$ to make the 4 case line up exactly).